On the Techniques of Clock Extraction and Oversampling

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Overview

Introduction

Two techniques for data recovery from serial streams:

- Clock extraction:
  - Phase-locked loop for placing sampling time point near-optimally within the bit time interval.
  - Widely employed in the telecommunications field.
- Oversampling
  - Free-running internal clock.
  - Multiple sampling of the received symbols.
  - Simple, low-cost “digital” technique.

Introduction

Probabilistic analysis:

- Simple differential signal model including additive noise and timing uncertainty (skew, jitter).
- Analytical expressions for the probability of error resulting from decision at the ideal sampling receiver.
- Majority decision rule for 3× oversampling.
- Quantitative results through numerical integration.
- Conclusive answer which technique is “better” not expected, but analysis displays some of the inherent trade-offs.

Signal model

Noisy signal representing a single received message $m = 0, 1$:

$$u(t \,|\, m) = s(t - \tau \,|\, m) + n(t) \tag{1}$$

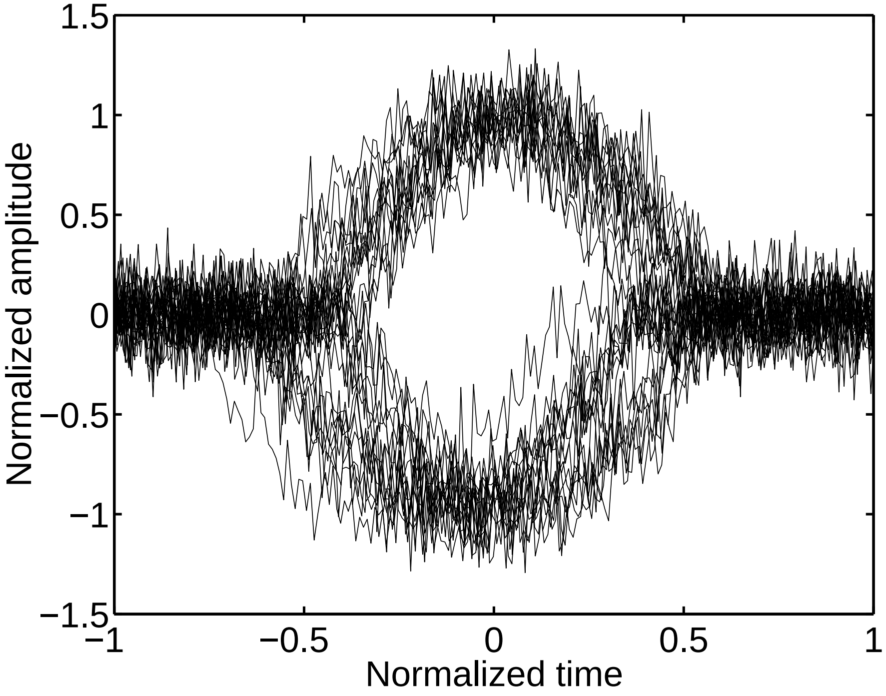
with

$$s(t \,|\, m) = \begin{cases} -A\,, & t = 0\,, \;\; m = 0 \\ A\,, & t = 0\,, \;\; m = 1 \\ 0\,, & |t| \geq T/2 \end{cases} \tag{2}$$

and

$$s(t \,|\, m) = -s(t \,|\, 1 - m) \tag{3}$$

Signal model



Realizations of a noisy differential signal
with time uncertainty ($S/N = 15\,$dB, $\sigma_\tau/T = 0.1$).

Probabilistic analysis

Probability of error for one sample taken at $t = t_0$:

$$\begin{aligned} P_{\mathrm{err}}(t_0) \;=\; & P[\,m=0\,]\,P[\,u(t_0\,|\,0) > 0\,] \\ & + P[\,m=1\,]\,P[\,u(t_0\,|\,1) < 0\,] \end{aligned} \tag{4}$$

Assuming equally probable messages,

$$P[\,m=0\,] = P[\,m=1\,] = \frac{1}{2} \tag{5}$$

and that $n(t)$ is evenly distributed about zero, we have

$$P_{\mathrm{err}}(t_0) = P[\,n(t_0) > s(t_0 - \tau\,|\,1)\,] \tag{6}$$

Probabilistic analysis

Spelling out in terms of probability density functions:

$$P_{\rm err}(t_0) = \int\limits_{-\infty}^{\infty} dy\, f_\tau(y) \int\limits_{s(t_0-y\,|\,1)}^{\infty} dx\, f_{n(t_0)}(x) \tag{7}$$

In the absence of any time uncertainty:

$$\hat{P}_{\rm err}(t_0) = \int\limits_{s(t_0|\,1)}^{\infty} dx\, f_{n(t_0)}(x) \tag{8}$$

For stationary noise $n(t)$:

$$P_{\rm err}(t_0) = f_\tau(t_0) * \hat{P}_{\rm err}(t_0) \tag{9}$$

Probabilistic analysis

Stationary Gaussian noise with standard deviation σ_n:

$$P_{\mathrm{err}}(t_0) = \frac{1}{2} - \frac{1}{2} \int\limits_{-T/2}^{+T/2} dy\, f_\tau(t_0 - y)\, \mathrm{erf}\left[\frac{s(y \,|\, 1)}{\sigma_n \sqrt{2}}\right] \tag{10}$$

For Gaussian jitter with mean μ_τ and standard deviation σ_τ:

$$P_{\mathrm{err}}(t_0) = \frac{1}{2} - \frac{1}{\sigma_\tau \sqrt{8\pi}} \int\limits_{-T/2}^{+T/2} dy\ \exp\left[-\frac{(t_0 - y - \mu_\tau)^2}{2\sigma_\tau^2}\right]\ \mathrm{erf}\left[\frac{s(y \,|\, 1)}{\sigma_n \sqrt{2}}\right] \tag{11}$$

Special case of no time uncertainty gives the familiar result:

$$\hat{P}_{\mathrm{err}}(t_0) = \frac{1}{2} - \frac{1}{2}\, \mathrm{erf}\left[\frac{s(t_0 \,|\, 1)}{\sigma_n \sqrt{2}}\right] \tag{12}$$

Clock extraction

General case:

$$P_{\text{err}}^{\text{ce}} = \int\limits_{-\infty}^{\infty} dy\, f_\tau(y) \int\limits_{s(-y\,|\,1)}^{\infty} dx\, f_{n(0)}(x) \tag{13}$$

Stationary case:

$$P_{\text{err}}^{\text{ce}} = \Big[f_\tau(t_0) * \hat{P}_{\text{err}}(t_0) \Big]_{t_0=0} \tag{14}$$

Gaussian case:

$$P_{\text{err}}^{\text{ce}} = \frac{1}{2} - \frac{1}{\sigma_\tau \sqrt{8\pi}} \int\limits_{-T/2}^{+T/2} dy\, \exp\left[-\frac{(y+\mu_\tau)^2}{2\sigma_\tau^2} \right] \operatorname{erf}\left[\frac{s(y\,|\,1)}{\sigma_n \sqrt{2}} \right] \tag{15}$$

Oversampling

Majority decision for 3× oversampling (stationary, white noise):

$$\begin{aligned} P_{\mathrm{err}}^{\mathrm{os}} \; = \; \Big\{ f_\tau(t_0) * \Big[& \hat{P}_{\mathrm{err}}^{-}(t_0)\, \hat{P}_{\mathrm{err}}(t_0)\, [1 - \hat{P}_{\mathrm{err}}^{+}(t_0)] \\ & + [1 - \hat{P}_{\mathrm{err}}^{-}(t_0)]\, \hat{P}_{\mathrm{err}}(t_0)\, \hat{P}_{\mathrm{err}}^{+}(t_0) \\ & + \hat{P}_{\mathrm{err}}^{-}(t_0)\, [1 - \hat{P}_{\mathrm{err}}(t_0)]\, \hat{P}_{\mathrm{err}}^{+}(t_0) \\ & + \hat{P}_{\mathrm{err}}^{-}(t_0)\, \hat{P}_{\mathrm{err}}(t_0)\, \hat{P}_{\mathrm{err}}^{+}(t_0) \Big] \Big\}_{t_0=0} \end{aligned} \tag{16}$$

where

$$\hat{P}_{\mathrm{err}}^{\pm}(t_0) = \hat{P}_{\mathrm{err}}(t_0 \pm T/3) \tag{17}$$

Collecting terms:

$$\begin{aligned} P_{\mathrm{err}}^{\mathrm{os}} \; = \; \Big\{ f_\tau(t_0) * \Big[& \hat{P}_{\mathrm{err}}^{-}(t_0)\, \hat{P}_{\mathrm{err}}(t_0) \\ & + \hat{P}_{\mathrm{err}}(t_0)\, \hat{P}_{\mathrm{err}}^{+}(t_0) + \hat{P}_{\mathrm{err}}^{-}(t_0)\, \hat{P}_{\mathrm{err}}^{+}(t_0) \\ & - 2\, \hat{P}_{\mathrm{err}}^{-}(t_0)\, \hat{P}_{\mathrm{err}}(t_0)\, \hat{P}_{\mathrm{err}}^{+}(t_0) \Big] \Big\}_{t_0=0} \end{aligned} \tag{18}$$

Numerical example

Sinusoidal pulse shape:

$$s(t\,|\,1) = A\,\cos\frac{\pi}{T}\,t\;, \quad |t| < \frac{T}{2} \tag{19}$$

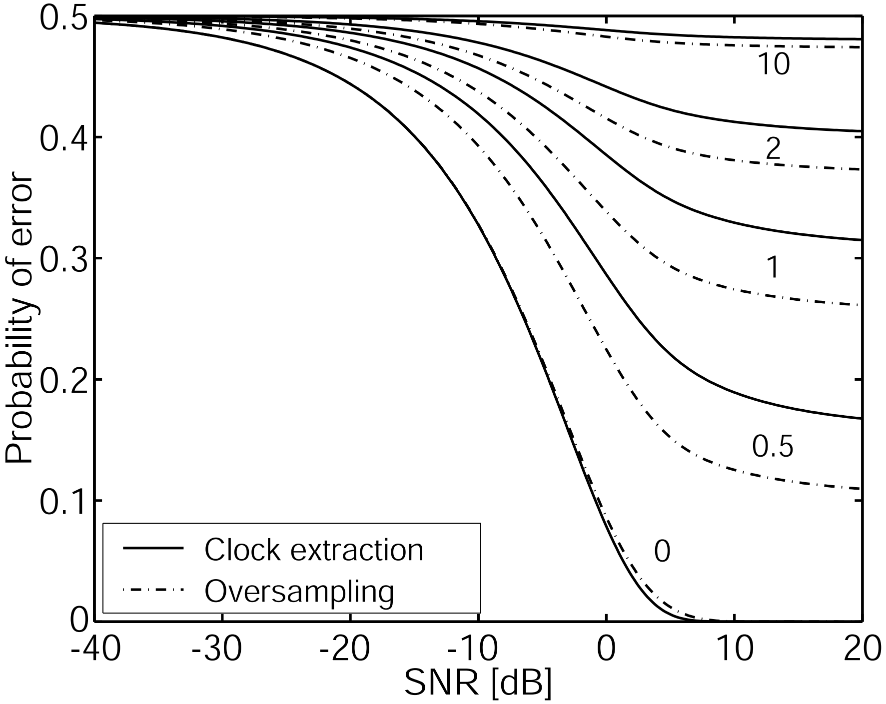
so that

$$\frac{S}{N} = \frac{A^2}{2\,\sigma_n^2} \tag{20}$$

with the signal-to-noise ratio:

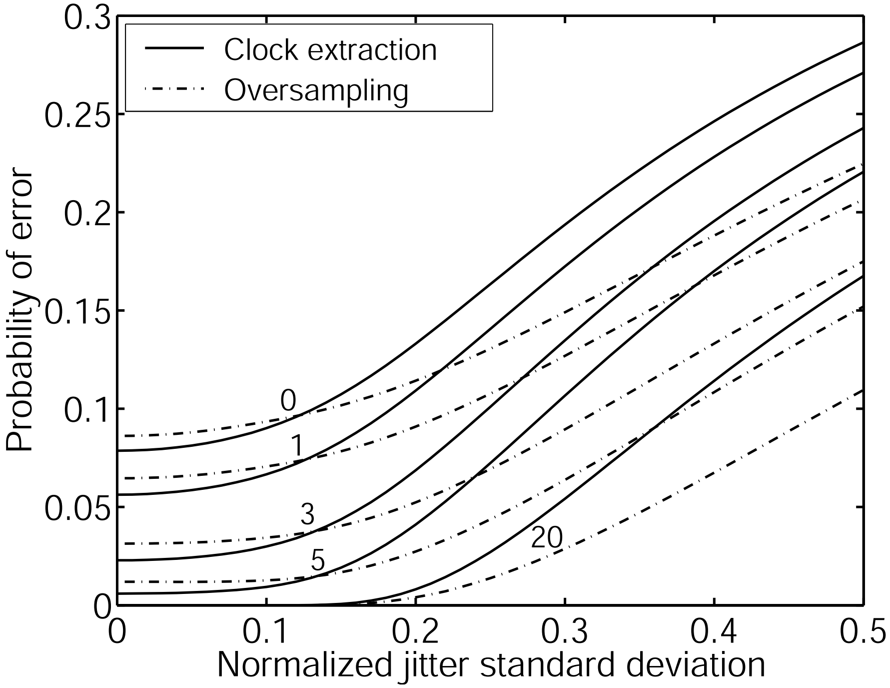
$$\frac{S}{N} = \frac{1}{\sigma_n^2\,T} \int\limits_{-T/2}^{T/2} dt\, s^2(t\,|\,m) \tag{21}$$

Numerical results



Probability of error for clock extraction and oversampling versus signal-to-noise ratio ($\sigma_\tau/T =$ 0, 0.5, 1, 2, 10).

Numerical results



Probability of error for clock extraction and oversampling versus jitter standard deviation ($S/N =$ 0, 1, 3, 5, 20 dB).

Conclusion

- Analytical formulations for the probability of error when sampling under the conditions of additive noise and sampling time uncertainty (skew, jitter).
- Numerical results for the comparison of oversampling and clock extraction.
- Which system yields lower bit error rates is critically dependent on the performance of the PLL in the clock extraction scheme.
- For the majority decision rule, multiple sampling does not help in the case of small sampling time uncertainty.